THE ESTIMATION OF DAMAGE STATUS AND FRAGMENT SIZE DISTRIBUTION FOR MINING AND TUNNELING APPLICATIONS

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Abstract

Rock failure widely exists in geotechnical engineering, particularly in tunneling and underground mining. Accurate estimation of fragment size distribution not only can ensure the safety and efficiency of engineering projects but is also helpful to save on transportation expenses and avoid costs caused by secondary fragmentation. This research proposes a method to estimate the size distribution of rock fragmentation based on the self-similarity. In this paper, a combined use of fractal theory, elasto-plastic theory and energy conservation theory was adopted. By considering damage energy and size distribution, the fractal damage constitutive model is proposed. In this model, fragment size, damage state and fractal dimension are three main influencing factors. To verify this model, red sandstone was selected as a case study. By fitting the stress-strain curves and quantity-frequency curves, the brittle index and fractal dimension were calculated. Through utilizing the method proposed in this research, the damage status and fragment size of jointed rock mass and collapsed roof in goaf can be estimated. Eventually, implementation of the estimator model would support the attempts towards autonomous operations and vision-based monitoring approaches.

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1. Introduction

The excavation and blasting in quarrying, underground mining, tunneling and other geotechnical engineering activities will cause creation and propagation of fracture inside rock mass. Rock fragmentation is widespread in rock engineering [1], [2]. The mechanical properties of the broken rock are closely related to the size distribution of the fragments. Many research results showed that the difference in size distribution of fragments significantly affected the macro mechanical behavior of broken rocks, such as stress-strain curves, shear response, etc. [3], [4].

Moreover, the engineering construction efficiency is also affected by the size distribution of fragments. Take tunneling as an example, the size distribution of broken rocks is the key factor to determine the transport mode and efficiency. Research results of Rehman [5], [6] and Ma [7] showed that the rock transportation could account for 20% to 40% of the whole project time, depending on the rock breaking method, rock mechanical properties and geological conditions.

If the fragment size is too large, it requires high-power conveyor and may need secondary crushing to reach the transportation standard. It also causes the growth of project costs. On the other hand, small size fragmentation increases the cost of rock breaking. The estimation of rock fragment distribution is a basis of tunnel construction and underground mining design. Screening is a direct way to obtain the size distribution of fragments and is also considered as a reliable method [8]. The test equipment includes vibrating screen, weighing device, etc. However, the result obtained by this method is only a sample value, which requires repeated tests. Meanwhile, sampling methods must also comply with standard requirements to reduce the test error [9].

The methods based on in-situ are time-consuming and in-situ test results are significantly affected by the geological conditions limited to the specific spot of measurement [10], [11]. In order to reduce the error, in-suit testing could be supported by the combined use of mechanical tests and theoretical analysis. Scholars proposed many theoretical methods to estimate the size distribution of rock fragments. Among these methods, the Rosin-Rammler distribution is most widely used [12]. Recently developed Swebrec Function [13] has improved performance on representing the fragmented rock sizes both in the fine and coarse conditions. Sanchidrián [14] calculated size-prediction errors in coarse, central, fines and very fines zones and the extended Swebrec was found the best function to fit the data.

The rock fragmentation caused by tunneling equipment or ground stress is different from that generated by blasting. It is closer to the failure under static load. There are also significant differences in the size distribution of the fragments between these methods. Mandelbrot popularized the concept of fractal theory in 1975 and since then this theory has been widely used to study the fragmentation of coal and rock mass and the self-similarity of fragments in the process of breaking [15], [16]. Using fractal theory to study the size distribution of rock fragments, particularly for the brittle formations is promising [17], [18]. In this research, based on the relations between damage, energy and fractal dimension, the estimation of the size distribution of rock fragments is studied.

2. Background of fractal theory

Fractal geometry focuses on certain irregular curves with self-similarity which refers to the feature that a superstructure is resembled by a substructure [19]. During the damage process, discontinuities or cracks are formed in the rock. Based on the size, discontinuities can be divided into three classes: macro-crack, meso-crack and micro-crack. Macro-cracks are formed by the propagation and nucleation of meso-cracks and micro-cracks. The development of crack cuts the rock into blocks and results in the jointed and fractured rock mass. From the view of dimension, rock fragmentation is a process that large rock mass breaks into small blocks and is further crushed into much smaller pieces. Based on the fractal theory, the size distribution of rock fragments and the morphology of crack both have the property of self-similarity [20]. Accordingly, the following equation can be used to calculate the fractal dimension (D_b).

$$N = C \cdot R^{-D_b} \tag{1}$$

where R is the equivalent fragment size, i.e. sieve diameter, N is the fragments count with the dimension of R or larger and C is the dimensional factor.

A large value of D_b indicates that the fragment has highly self-similarity and damage state. If D_b increases, the size of fragment decreases. The fractal dimension and size-frequency of fragment can be calculated as given in Equation (2).

$$N = N_0 \left(R / R_{\max} \right)^{-D_b} \tag{2}$$

where $R_{\max}$ is the maximum equivalent fragment size and N_0 is the number of fragments within the size range of $R_{\max}$. When D_b is greater than 1, the rock fragmentation degree is large.

According to Equation (2), through counting the number of fragments which matches the size requirement, the fractal dimension can be calculated. However, since the shape of rock fragments is an irregular polyhedron, the dimension is difficult to measure. Therefore, the quality-frequency is used to calculate Db according to Equation (3).

$$n = n_0 \left(M / M_{\max} \right)^{D_b^{'}} \tag{3}$$

where M is the quality of fragment; n is the number of fragment with larger quality than M, $M_{\max}$ is the maximum quality of the fragment, n_0 is the number of fragments which have the maximum quality and $D_b^{'}$ is fractal dimension of quality-frequency distribution. Since M is proportional to R_3, the relationship between fractal dimensions of size and quality can be calculated by the equation below [21].

$$D_b^{'} = 3D_b \tag{4}$$

3. Fractal failure of rock

3.1 Fractal damage model based on energy conservation

With the increase of load, the damage develops and results in fragmentation. The development of crack and the size of rock fragment have a close relationship. For a plastic rock, the damage and fragmentation gradually develop before and after the load reaches the peak stress. A brittle rock however, crushes within a very short range of strain when the applied stress reaches the strength σ_c. The strain for σ_c (ε_p) can be measured by the uniaxial compression test (UCT). For the brittle rock, it can be assumed that the damage only contains the development of fracture, without considering the rheology and fraction (pure damage). Based on the first law of thermodynamics, the relationship between the size of fragment and pure damage can be described as by Equation (5).

$$\frac{r}{R_0} = C \cdot D^{\frac{1}{D_b - 3}} \tag{5}$$

where R_0 is the size of rock before the test (in UCT, it equals to 100mm), r is the minimum equivalent dimension which stands for the fragment size with systemic self-similar characteristics, D is damage variable and C is dimensional constants.

3.2 Construction of damage constitutive model

Since fractal dimension and feature size of fragment change in different loading stages, the relationship between D and D_b can be constructed by the following equation.

$$D = f\left(r, D_b\right) \tag{6}$$

Equation (6) indicates that the size distribution of broken rock can be estimated when the damage state and the fractal dimension are known. Moreover, the damage state of engineering rock mass can be quantified according to Geological Strength Index (GSI) [22] which is based on the occurrence state of joints and cracks. The feature size can be acquired through metrical data. By substituting the parameters into Equation (6), the fractal characteristic of engineering rock mass can be obtained. D increases with crack development. To normalize D, the ratio of strains is used in the following equations.

$$D = \left(\frac{\varepsilon}{\varepsilon_s} \right)^n \tag{7}$$

$$\sigma = E \cdot (1 - D) \varepsilon' \tag{8}$$

where *n* is the brittle index of rock, *E* is the modulus of elasticity and ε' is the strain of rock under stress *σ*. By taking the derivative of *D*, the damage evolution equation can be obtained as given in Equation (9).

$$\dot{D} = \frac{n}{\varepsilon_s} \left(\frac{\varepsilon}{\varepsilon_s} \right)^{n-1} \tag{9}$$

The brittleness index of rock used in Equation (7) can be obtained by fitting the stress-strain curve. The damage model covers the fractal dimension and feature size of rock as defined by Equation (9). Generally, the damage and plastic deformation are the main reasons for energy dissipation after the failure. The above two processes consume the elastic strain energy stored in rock before and after peak strength. In post-peak stage, the relative movement of fragment along joints is the primary performance of plastic deformation. After this stage, rock is completely damaged and crack is sufficiently developed, so *D* equals to 1. Particularly for brittle rock (the value of *n* is large), the damage rarely develops after peak strength. According to the definition of pure damage, the relative slippage between discontinuity surfaces is ignored in this research. The pure damage can also be isolated from elasto-plastic damage through fitting the loading curve according to the damage constitutive model.

3.3 Description of n

The Rock brittleness index n describes the rate of stress decline of material after peak strength. According to Equation (7) and (8), the stress-strain curves with different *n* are shown in Figure 1.

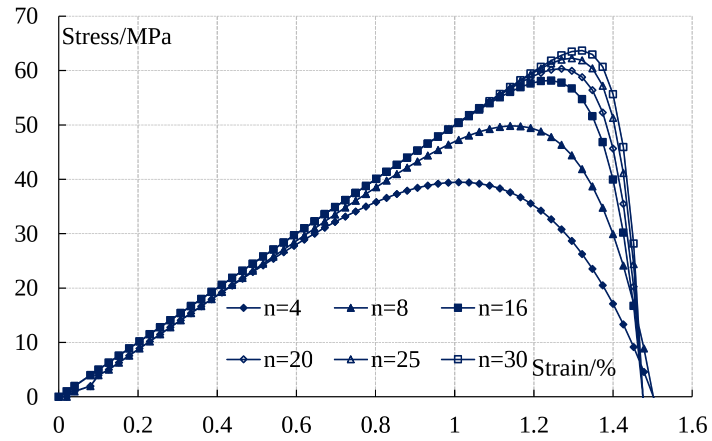

Figure 1. The stress-strain curves with different *n* values

The amount of strain after peak stress reduces with the increase of *n*. The peak stress and *n* have a positive relationship. Therefore, the energy used to crush rock decreases. Since it is less than the accumulated elastic strain energy, the difference between energy in rock before and after peak strength releases in the form of kinetic energy. This energy may result in dynamic disasters such as coal and rock outburst. Brittleness of rock causes insufficient crack extension and results in a large fractal dimension. Accordingly, n mainly determines the difference between ε_p and the complete failure strain of rock ε_c, as shown in Figure 2. As the brittleness of rock increases, ε_c-ε_p becomes smaller. Therefore, this curve can be used to qualitatively describe the brittleness of rock through the fragmental dimension-damage evolution curve. The quicker the curve declines in the post-peak zone, the stronger the brittleness of the rock is.

According to the stress-strain curve, *C* can be calculated. Red sandstone was used to verify the proposed model in this research due to its uniform properties. The specimens were drilled using a coring machine, and core dimensions were set as *Φ*50mm×100 mm.

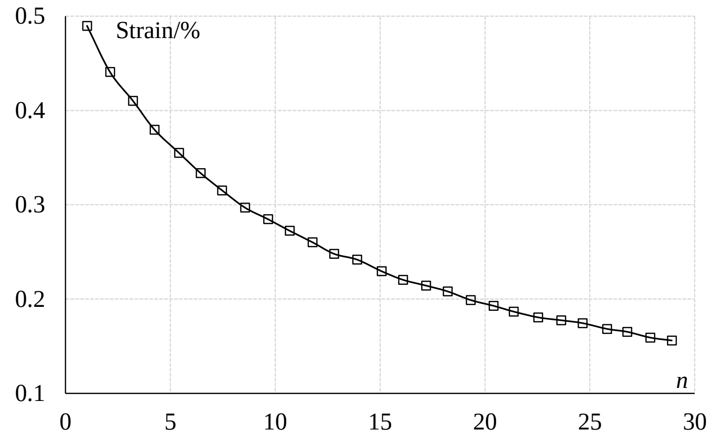

Figure 2. The influence of n on the difference between ε_p and ε_c

4. Experimental determination of *C*

4.1. Calculation of D_b

A servo-controlled electro-hydraulic rock mechanics testing system (MTS 815) was used to conduct the UCT of red sandstone. The fragments were classified by weight as shown in Figure 3.

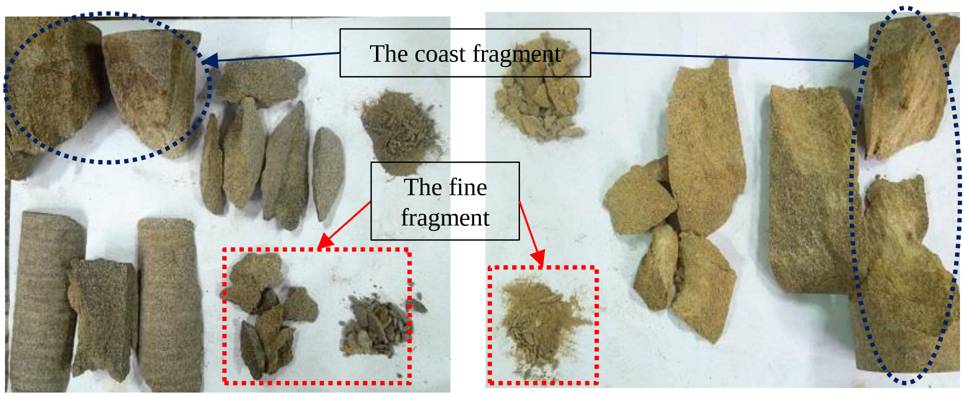

Figure 3. Rock fragments of red sandstone after UCT

Since the broken status of the specimen was mainly conjugate shear, the fragments in the ends and lateral of the specimen had larger weight than other areas. The number of fragments in different weight ranges was counted. Furthermore, the quality-frequency curves were obtained based on Equation (2) (Figure 4).

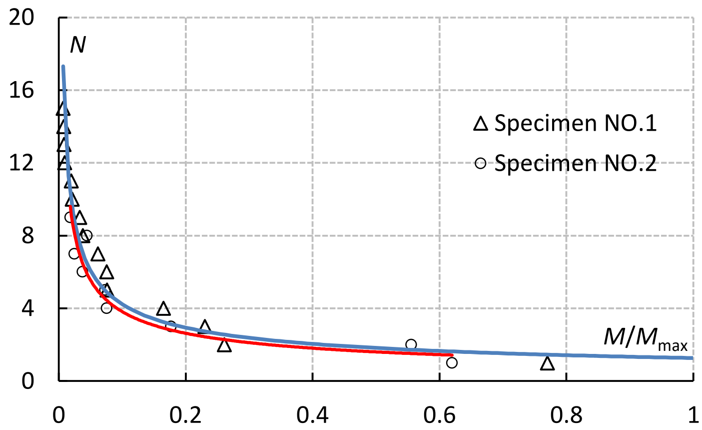

Figure 4. The quality-frequency distribution of red sandstone fragments

The dimension of sandstone fragments after UCT had evident statistical similarity. According to the curves, the quality of fragments showed a clear fractal characteristic. Through fitting the curves, the quality-frequency equation was be obtained as follows.

$$N = 1.267\left(\frac{M}{M_{\max}}\right)^{0.523}, \quad R^2 = 0.9387 \tag{10}$$

$$N = 1.105\left(\frac{M}{M_{\max}}\right)^{0.5239}, \quad R^2 = 0.9146 \tag{11}$$

According to Equation (4) and (5), the average D_b of the red sandstone is equal to 1.593. The relation between fragmental size and damage is illustrated in Figure 5. As the damage accumulated during the whole loading stage, the slope of the curve varied in different stages. During the early phase, the size of fragment decreased rapidly. For brittle rocks, such as sandstone, basalt, marble, etc., the fracture and fragmentation of rock developed quickly in this stage. After that, the speed of fragmentation decreased, but the damage developed quickly. Therefore, the two stages could be named as the crush and damage stages. Consequently, the size of fragment had a close relationship with damage stage and the link between fractal dimension and damage could be established.

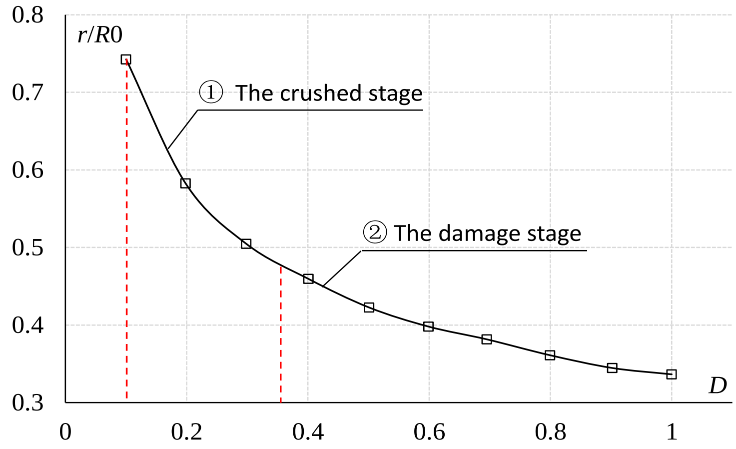

Figure 5. The relationship between fragment dimension and damage state

In UCT, the main causes of rock damage are the development of tensile and shear fractures. In the test, when the brittle red sandstone breaks, the failure of the specimen is dominated by the through longitudinal shear cracks ("X" type breakage). This limits the development of other small cracks. Therefore, the fragments size will be uneven. The increase of D means a high degree of rock fragmentation and a decrease in the size of fragments. It provides the possibility for the increase of D_b.

4.2. Calculation of C

In order to determine the value of the dimensional factor using the stress-strain curve, several calculating points on are required to be selected and fitted. The complete stress-strain curve for pure damage can be obtained according to Equation (6) and (8). And then, n can be obtained from the curve fitting. Since the after-peak strain is small for brittle rock, it is difficult to select the calculating points and to construct the link between D_b and C.

For brittle rocks, before reaching Uniaxial Compressive Strength (UCS), a large amount of rock elastic potential energy is accumulated inside. After the main crack of the rock is formed, the fragments consume elastic energy in the form of kinetic energy. Therefore, through mechanical testing, it is difficult to obtain the development process of post-peak stress. Hajiabdolmajid suggested that within the post-peak stage, the stress of brittle rock should fall from the UCS within the strain of 5% ε_c. According to Equation (9), the minimum value of n for brittle rocks is 25.7. In order to meet the needs of fitting calculation, in this research, the specific damage status points are chosen from the fitted curves as shown in Figure 3.

According to the complete stress-strain curve (Figure 8), the value of ε_c and ε_s were 1.412% and 1.383% respectively and n was equal to 38.22. By measuring and substituting r into the calculation, the value of C was determined as 0.33.

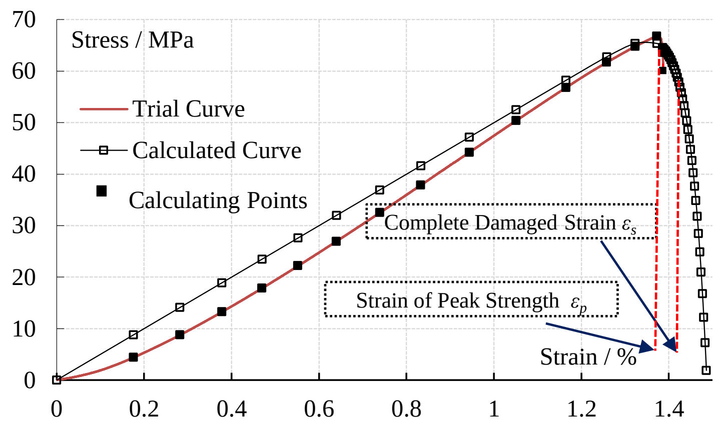

Figure 8. Complete stress-strain curve of specimens

Finally, based on Equation (5), the fractal distribution of sandstone can be expressed using the following equation.

$$\frac{r}{R_0} = 0.33D^{-0.7101} \tag{12}$$

Equation (12) demonstrates the size-damage fractal evolution of sandstone. Based on this equation, the size of the fragment can be predicted when the damage state is obtained.

5. Conclusions

In this research, in order to quantitatively predict the size distribution of rock fragments, the fractal damage constitutive model is proposed based on fractal theory. It mainly consists of three major factors: fragment dimension, damage state and fractal dimension. In this model, the damage state is related to strain and brittleness index. The fractal dimension is obtained by fitting the quality-frequency distribution curve of fragments. According to the stress-strain curve, other material constants can also be obtained. In order to solve the parameters, the red sandstone was selected as a case study. Through mechanical experiments and calculations, the damage equation of sandstone was established. By using this method, the damage state and fragment size can be estimated.

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