SWITCHED H_2 STATE-FEEDBACK CONTROL FOR POLYTOPIC SYSTEMS BASED ON DWELL TIME

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Abstract

This paper introduces a H_2 sub-optimal approach to design state feedback controllers for switched system with parameter uncertainty. Minimum dwell time stability analysis techniques are revisited and combined with the H_2-optimal control to obtain a robust state feedback controller for switched systems. Two kinds of Lyapunov functions have been used to find a switched controller, the time varying and constant controller gains have been given for the switched system. The proposed approaches have been illustrated by means of examples.

Keywords: H_2-optimal control, Switched Systems, Lyapunov function, Minimum Dwell time.

1. Introduction

Switched systems appear in many applications such as auto pilot design, heating systems, automatic transmissions, communication networks, DC-DC converters automotive engine control, and so on. Such switched control systems may consist of locally designed controllers switched according to a logic rule designed to meet the desired control objectives [1].

In the last years, the analysis and synthesis of switched systems have got more attention because of their importance in both theory and applications. For the stability analysis and control design of switched systems, many important methods have been investigated by

several researchers so far (see [2–14] and references therein). For the stability analysis issue, various techniques have been mentioned such as common quadratic Lyapunov function method, multiple Lyapunov functions method, average and minimum dwell-time methods [9–17]. In most cases, common quadratic Lyapunov method does not provide a solution for given switched systems due to its conservativity. Multiple Lyapunov function method provides the asymptotic stability of the switched system for any switching signal, if the all subsystems are stable and the time interval between two consecutive switching instants is sufficiently large. From this, minimum dwell time switching theories have been developed in recent works to guarantee stability of switched linear system and obtain minimum dwell time between two consecutive switching [7–10].

Optimal control methods, which are part of the modern control theory, play an important role in many feedback control design. Optimal control focuses on the minimization of a performance index (PI) that depends on the system matrices. The PI might contain, for example, information of control effort, information of operating error, or any other features which are necessary to the user of control system. The main purpose of optimal control theory is to determine a control signal that satisfies the terminal state constraints and also minimize the PI [18].

In the current work, we address the H_2-optimal control problem, and state feedback controller synthesis for switched systems with dwell time. In this case, the switched system parameters have polytopic uncertainty. In [5], conservatism of the robust H_∞ design is reduced by using a piecewise linear quadratic Lyapunov function in time. In [6], stability and stabilization problem of switched linear systems with dwell time are solved by using the same function. In [8], this function is also used to find and minimize the L_2-gain of uncertain switched linear systems with dwell time. Using the same function, robust filter is designed in [19]. We apply here the same Lyapunov function to H_2-optimal control in order to minimize sensitivity of the system output to a zero mean white noise input.

This paper is as follows: A minimum dwell time stability analysis is presented in Section 2. The new approach to compute the robust state feedback switched controller is

given in Section 3. There illustrative examples about the proposed approaches are given in Section 4. Finally, Section 5 gives conclusions.

Notation: $Tr(.)$ denotes the trace function for square matrices. The hermitian operator $He\{.\}$ is defined as $He\{A\} = A + A^T$. For symmetric matrices, $P > 0 \, (\geq 0)$ indicates that P is positive definite (semi-definite). A symmetric matrix $\begin{bmatrix} P & Q \\ Q^T & R \end{bmatrix}$ is denoted by $\begin{bmatrix} P & Q \\ * & R \end{bmatrix}$.

$\mathfrak{R}^n$ stands for the n dimensional Euclidean space. $\mathfrak{R}^{p \times q}$ is the set of all $p \times q$ real matrices. N is set of natural numbers.

2. Preliminaries

A switched system with polytopic uncertainty is defined as

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{u,\sigma(t)}u(t) + B_{w,\sigma(t)}w(t), \quad \forall t \geq 0 \quad \text{and} \quad x(0) = x_0. \tag{1}$$

Where $x(t) \in \mathfrak{R}^n$ is the state, x_0 is initial condition, $u(t) \in \mathfrak{R}^p$, $w(t) \in \mathfrak{R}^r$ is the exogenous input and $\sigma(t)$ represents the switching rule between system matrices $A_i \in \mathfrak{R}^{n \times n}$ $i = 1, \ldots, M$. The uncertain system matrices are assumed to reside within the following polytope:

$$A_i = \sum_{j=1}^N \eta_j A_i^{(j)}, \qquad B_{u,i} = \sum_{j=1}^N \eta_j B_{u,i}^{(j)},$$
$$B_{w,i} = \sum_{j=1}^N \eta_j B_{w,i}^{(j)}, \quad \sum_{j=1}^N \eta_j = 1, \ \eta_j \geq 0, \tag{2}$$

where j is the index of the polytope vertices and N denotes number of polytope vertices.

Definition 1: Dwell time is a time-interval between two switching instants $(t_1, t_2, \ldots)$. It is assumed to be equal or greater than a minimum dwell time $T_d, (t_{l+1} - t_l \geq T_d$ for $\forall l \geq 1)$.

Definition 2: Consider the index of the initial subsystem, $i_0 = \sigma(0)$ and the commutation instants $t_1, t_2, \ldots$, where $t_{l+1} - t_l \geq T_d$, $\forall l \geq 1$ and $l \in \mathrm{N}$. If the minimum dwell time between two switching instants is divided into H equal sampling time intervals, then

$t_{l,h}$ is defined as $t_{l,h} = t_l + hT_d/H$ for $h = 0,\ldots,H$, and $t_l = t_{l,0}$. The time-varying Lyapunov function is defined as

$$V(t) = x^T(t)P_{\sigma(t)}(t)x(t), \tag{3}$$

where $P_{\sigma(t)}(t)$ which is in $\{P_1(t),\ldots,P_M(t)\}$ according to $\sigma(t)$, defined as:

$$P_i(t) = \begin{cases} P_{i,h} + (P_{i,h+1} - P_{i,h})\dfrac{(t-t_{l,h})}{T_d/H} & t \in [t_{l,h}, t_{l,h+1}), \\ P_{i,H} & t \in [t_{l,H}, t_{l+1,0}), \\ P_{i_0,H} & t \in [0, t_1), \end{cases} \tag{4}$$

where $h = 0,\ldots,H-1$, the number of the switching is defined as $l \in \mathbb{N}$ and where H is given a positive integer. This definition shows that the matrices $P_i(t)$ are constant and equal to $P_{i_0,H}$ before the first switching instant. During the minimum dwell time, the matrices $P_i(t)$ change linearly. After the minimum dwell time, and before the following switching instant, the matrices $P_i(t)$ are constant and equal to $P_{i,H}$.

Based on the time-varying Lyapunov function, the following lemma deals with the stability of a switched system with polytopic uncertainty.

Lemma 1 ([6]): For a given $T_d > 0$, if there exist a set of positive definite matrices $P_{i,h} > 0$, $i = 1,\ldots,M$, $h = 0,\ldots,H$ that satisfy the following LMI's for all $i = 1,\ldots,M$ and $j = 1,\ldots,N$:

$$\frac{(P_{i,h+1} - P_{i,h})}{T_d/H} + He\{P_{i,h}A_i^{(j)}\} < 0, \tag{5a}$$

$$\frac{(P_{i,h+1} - P_{i,h})}{T_d/H} + He\{P_{i,h+1}A_i^{(j)}\} < 0, \tag{5b}$$

where $h = 0,\ldots,H-1$ and

$$He\{P_{i,H}A_i^{(j)}\} < 0, \tag{5c}$$

$$P_{i,H} - P_{s,0} \geq 0, \forall s \in \{1,\ldots,M\} \text{ and } s \neq i \tag{5d}$$

then the system (1) with the uncertainty (2) is globally asymptotically stable for any switching rule with dwell time that is greater than the minimum dwell time, T_d (see [6], for detailed proof).

Note that, before the first switching, the decreasing of the Lyapunov function is proved by the condition (5c). During the minimum dwell time, the conditions in (5a-b) prove that the Lyapunov function is decreasing. Then, the condition in (5c) proves that $V(t)$ is decreasing, after the minimum dwell time and before the next switching. During the switching instants, the Lyapunov function is proved to be non-increasing by the condition in (5d) [6].

The Lyapunov function (3) uses the same Lyapunov matrix (4) for all vertices of the subsystems. Hence, it is more conservative with regards to the stability conditions. This conservatism issue is reduced by using the *time-varying and parameter (vertex) dependent Lyapunov function* [6].

Definition 3: Time-varying and parameter dependent Lyapunov function is

$$V(t) = x^T(t)P_{\sigma(t)}(t)x(t), \quad P_{\sigma(t)}(t) = \sum_{j=1}^{N}\eta_j P_{\sigma(t)}^{(j)}(t), \quad \sum_{j=1}^{N}\eta_j = 1, \eta_j \geq 0, \tag{6}$$

where $P_{\sigma(t)}^{(j)}$ which is in $\{P_1^{(j)}(t),\ldots,P_M^{(j)}(t)\}$ according to $\sigma(t)$, defined as:

$$P_i^{(j)}(t) = \begin{cases} P_{i,h}^{(j)} + \left(P_{i,h+1}^{(j)} - P_{i,h}^{(j)}\right)\dfrac{(t - t_{l,h})}{T_d / H} & t \in [t_{l,h}, t_{l,h+1}), \\[2mm] P_{i,H}^{(j)} & t \in [t_{l,H}, t_{l+1,0}), \\[2mm] P_{i_0,H}^{(j)} & t \in [0, t_1), \end{cases} \tag{7}$$

where $h = 0,\ldots,H-1, \ j = 1,\ldots,N,$ the number of the switching is defined as $l \in N$ and H is given a positive integer. Note that, the difference between (3) and (6) is that Lyapunov matrices in (6) depend on the sub-polytopes vertices [6].

Based on the time-varying and parameter dependent Lyapunov function, the following lemma deals with the stability of a switched system with polytopic uncertainty.

Lemma 2 ([6]): For a given $T_d > 0$, if there exist a set of matrices $S_{i,h}, G_{i,h}$ and positive definite matrices $P_{i,h}^{(j)} > 0$, $i = 1, \ldots, M$, $h = 0, \ldots, H$, $j = 1, \ldots, N$, such that, for all $i = 1, \ldots, M$, and $j = 1, \ldots, N$ the following conditions hold

$$\begin{bmatrix} \dfrac{\left(P_{i,h+1}^{(j)} - P_{i,h}^{(j)}\right)}{T_d / H} + He\left\{S_{i,h} A_i^{(j)}\right\} & P_{i,h}^{(j)} - S_{i,h} + A_i^{(j)^T} G_{i,h}^T \\ * & -G_{i,h}^T - G_{i,h} \end{bmatrix} < 0, \tag{8a}$$

$$\begin{bmatrix} \dfrac{\left(P_{i,h+1}^{(j)} - P_{i,h}^{(j)}\right)}{T_d / H} + He\left\{S_{i,h+1} A_i^{(j)}\right\} & P_{i,h+1}^{(j)} - S_{i,h+1} + A_i^{(j)^T} G_{i,h+1}^T \\ * & -G_{i,h+1}^T - G_{i,h+1} \end{bmatrix} < 0, \tag{8b}$$

where $h = 0, \ldots, H-1$ and

$$\begin{bmatrix} He\left\{S_{i,H} A_i^{(j)}\right\} & P_{i,H}^{(j)} - S_{i,H} + A_i^{(j)^T} G_{i,H}^T \\ * & -G_{i,H}^T - G_{i,H} \end{bmatrix} < 0, \tag{8c}$$

$$P_{i,H}^{(j)} - P_{s,0}^{(j)} \geq 0, \quad \forall s \in \{1, \ldots, M\}, \text{ and } s \neq i \tag{8d}$$

then the system (1) with an uncertainty (2) is globally asymptotically stable for any switching rule with dwell time that is greater than minimum dwell time, T_d (see [6], for detailed proof).

In this work, Lemmas 1 and 2 are used to find a stabilizing switched state feedback controller as in following section.

3. Robust Controller Design

In this section, we combine the standard H_2-optimal control and the minimum dwell time methods mentioned in the previous section. In addition, it allows us to design the state feedback H_2-optimal controller for the switched system with polytopic parameter uncertainties. Here, the sensitivity of the system output to a zero mean white noise input w is minimized by using H_2 optimization technique.

Lemma 3 (H_2-gain [21]): Consider a linear time-varying system,

$$\dot{x}(t) = A(t)x + B_w(t)w, \quad z(t) = C(t)x, \tag{9}$$

where w is the exogenous input and z is the system output. If there exist symmetric matrices, $Q(t) > 0$ and Z, then H_2-gain of the system given in (9) is less than γ, if the following conditions hold:

$$
Tr(Z) < \gamma^2, \quad \begin{bmatrix} Z & B_w^T(t) \\ * & Q(t) \end{bmatrix} > 0,
$$

$$
\begin{bmatrix} -\dot{Q}(t) + He\{A(t)Q(t)\} & Q(t)C^T(t) \\ * & -I \end{bmatrix} < 0, \forall t \tag{10}
$$

For a switched system, Q is defined as: $Q_{\sigma(t)}(t)$ with $\sigma(t) \in \{1,2,\ldots,M\}$ and where $Q_i(t)$ is defined as:

$$
Q_i(t) = \begin{cases} Q_{i,h} + (Q_{i,h+1} - Q_{i,h})\dfrac{(t-t_{l,h})}{T_d/H} & t \in [t_{l,h}, t_{l,h+1}), \\[2mm] Q_{i,H} & t \in [t_{l,H}, t_{l+1,0}), \\[2mm] Q_{i_0,H} & t \in [0,t_1), \end{cases} \tag{11}
$$

where $h = 0,\ldots,H-1$, the switching index is defined as $l \in \mathbb{N}$ and H is given a positive integer. Using Lemmas 1 and 3, the following Theorem is given for the state feedback controller gain.

Theorem 1: For a given $T_d > 0$, if the set of matrices $L_{i,h}$ of compatible size and the collection of symmetric matrices, Z and $Q_{i,h} > 0$, $i = 1,\ldots,M$, $h = 0,\ldots,H$ satisfy the following LMI's for all $i = 1,\ldots,M$, and $j = 1,\ldots,N$:

$$\begin{bmatrix} Z & B_{w,i}^{(j)T} \\ * & Q_{i,h} \end{bmatrix} > 0, \ \forall h \in \{0,\ldots,H\},$$

$$\begin{bmatrix} \dfrac{\left(Q_{i,h}^{(j)} - Q_{i,h+1}^{(j)}\right)}{T_d / H} + He\left\{A_i^{(j)}Q_{i,h} + B_{u,i}^{(j)}L_{i,h}\right\} & Q_{i,h}C_i^{(i)T} + L_{i,h}{}^T D_{u,i}^{(j)T} \\ * & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \dfrac{\left(Q_{i,h}^{(j)} - Q_{i,h+1}^{(j)}\right)}{T_d / H} + He\left\{A_i^{(j)}Q_{i,h+1} + B_{u,i}^{(j)}L_{i,h+1}\right\} & Q_{i,h+1}C_i^{(i)T} + L_{i,h+1}{}^T D_{u,i}^{(j)T} \\ * & -I \end{bmatrix} < 0,$$

where $h = 0,\ldots,H-1$ and

$$\begin{bmatrix} He\left\{A_i^{(j)}Q_{i,H} + B_{u,i}^{(j)}L_{i,H}\right\} & Q_{i,H}C_i^{(i)T} + L_{i,H}{}^T D_{u,i}^{(j)T} \\ * & -I \end{bmatrix} < 0,$$

$Q_{i,H} - Q_{s,0} \leq 0, \ \forall s \in \{0,\ldots,M\}$ and $s \neq i$

then there exists a state-feedback regulator $K_{\sigma(t)}(t)$ which is in $\{K_1(t),\ldots,K_M(t)\}$ according

to $\sigma(t)$, defined as:

$$K_i(t) = \begin{cases} \overline{L}_{i,h}\overline{Q}_{i,h}^{-1} & t \in [t_{l,h}, t_{l,h+1}), \\ L_{i,H}Q_{i,H}^{-1} & t \in [t_{l,H}, t_{l+1,0}), \\ L_{i_0,H}Q_{i_0,H}^{-1} & t \in [0, t_1), \end{cases} \tag{12}$$

where $\overline{L}_{i,h} = L_{i,h} + \left(L_{i,h+1} - L_{i,h}\right)\left(t - t_{l,h}\right)H / T_d$ and $\overline{Q}_{i,h} = Q_{i,h} + \left(Q_{i,h+1} - Q_{i,h}\right)\left(t - t_{l,h}\right)H / T_d$,

such that the closed-loop system (system (1) with (12) and uncertainty (2)) is quadratically

stable for any switching rule with a dwell time that is greater than the minimum dwell time,

T_d and such that the H_2-gain from w to z is less than or equal to $\gamma = \sqrt{Tr(Z)}$. To get the

best upper bound, we need to minimize $Tr(Z)$. Note that this theorem provides a state

feedback gain which is time varying during the dwell time.

In most cases, a time varying controller may not be desirable for the switched systems,

so a time invariant state feedback controller can be designed using the following lemma. The

parameter (vertex) dependent Lyapunov function (6) is used to reduce the conservatism in

Lemma 2. Hence, we combine with this lemma and H_2-optimal control approach in Lemma 3, and then design a time invariant state feedback controller for the switched system with polytopic parameter uncertainty.

The time-varying and parameter dependent positive definite matrix, $Q_{\sigma(t)}^{(j)}$ which is in $\{Q_1^{(j)}(t),\ldots,Q_M^{(j)}(t)\}$ according to $\sigma(t)$, defined as:

$$Q_i^{(j)}(t) = \begin{cases} Q_{i,h}^{(j)} + \left(Q_{i,h+1}^{(j)} - Q_{i,h}^{(j)}\right)\dfrac{(t - t_{l,h})}{T_d / H} & t \in \left[t_{l,h}, t_{l,h+1}\right), \\ Q_{i,H}^{(j)} & t \in \left[t_{l,H}, t_{l+1,0}\right), \\ Q_{i_0,H}^{(j)} & t \in \left[0, t_1\right). \end{cases} \tag{13}$$

where $h = 0,\ldots,H-1$, $j = 1,\ldots,N$, the number of the switching is defined as $l \in \mathrm{N}$ and H is given a positive integer. Using Lemmas 2 and 3, the following Theorem is given for a time invariant state feedback controller gain.

Theorem 2: For given scalars $\beta > 0$ and $T_d > 0$, if the set of matrices, S_i and L_i are of compatible size and the collection of symmetric matrices, Z and $Q_{i,h}^{(j)} > 0$ $i = 1,\ldots,M$, $h = 0,\ldots,H$, $j = 1,\ldots,N$ satisfy the following LMI's for all $i = 1,\ldots,M$, and $j = 1,\ldots,N$:

$$\begin{bmatrix} Z & B_{w,i}^{(j)T} \\ * & Q_{i,h}^{(j)} \end{bmatrix} > 0, \ \forall h \in \left\{0,\ldots,H\right\},$$

$$\begin{bmatrix} \dfrac{\left(Q_{i,h}^{(j)} - Q_{i,h+1}^{(j)}\right)}{T_d / H} + \psi & S_i^T C_i^{(i)T} + L_i^T D_{u,i}^{(j)T} & Q_{i,h}^{(j)} - S_i^T + \beta(A_i^{(j)} S_i + B_{u,i}^{(j)} L_i), \\ * & -I & \beta(C_i^{(j)} S_i + D_{u,i}^{(j)} L_i) \\ * & * & -\beta(S_i^T + S_i) \end{bmatrix} < 0,$$

$$
\begin{bmatrix}
\dfrac{\left(Q_{i,h}^{(j)} - Q_{i,h+1}^{(j)}\right)}{T_d / H} + \psi & S_i^T C_i^{(i)T} + L_i^T D_{u,i}^{(j)T} & Q_{i,h+1}^{(j)} - S_i^T + \beta(A_i^{(j)}S_i + B_{u,i}^{(j)}L_i), \\
* & -I & \beta(C_i^{(j)}S_i + D_{u,i}^{(j)}L_i) \\
* & * & -\beta(S_i^T + S_i)
\end{bmatrix} < 0,
$$

where $h = 0, \ldots, H-1$ and

$$
\begin{bmatrix}
\psi & S_i^T C_i^{(i)T} + L_i^T D_{u,i}^{(j)T} & Q_{i,H}^{(j)} - S_i^T + \beta(A_i^{(j)}S_i + B_{u,i}^{(j)}L_i), \\
* & -I & \beta(C_i^{(j)}S_i + D_{u,i}^{(j)}L_i) \\
* & * & -\beta(S_i^T + S_i)
\end{bmatrix} < 0
$$

$Q_{i,H}^{(j)} - Q_{s,0}^{(j)} \le 0, \ \forall s \in \{0, \ldots, M\}$ and $s \ne i$

Here: $\psi = He\{A_i^{(j)}S_i + B_{u,i}^{(j)}L_i\}$, then there exists a state-feedback regulator defined as: $K_i = L_i S_i^{-1}$ such that the closed-loop system (system (1) with K_i and uncertainty (2)) is quadratically stable for any switching rule with a dwell time that is greater than the minimum dwell time, T_d and such that the H_2-gain from w to z is less than or equal to $\gamma = \sqrt{Tr(Z)}$. To get the best upper bound, we need to minimize $Tr(Z)$.

4. Examples

Example 1 (Uncertainty): Consider the system taken from [20] where the uncertainty is added to the subsystems

$$
A_1 = \begin{bmatrix} \lambda & 1 \\ -2 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} \lambda & 1 \\ -9 & -1 \end{bmatrix}, \quad |\lambda| \le \bar{\lambda} \tag{14}
$$

The stability of the system is proven by using Lemma 1 and Lemma 2. The minimum dwell time results for different uncertainty values are given in Table 1 and 2.

Tables 1 and 2 show that a less conservative result is found when H is increased. In addition, Lemma 2 always finds the smallest T_d. This proves that Lemma 2 gives less conservative results than Lemma 1 due to using parameter dependent Lyapunov function (6).

These results encourage us to use Lemma 2 in the controller design approach.

Table 1. Minimum dwell time, T_d results of Lemma 1

$H \setminus \lambda$	0	0.2	0.4	0.6	0.8
1	0.8538	1.2443	1.9696	3.8055	21.4936
2	0.7460	1.1324	1.7168	3.1325	14.8181
5	0.6727	1.0537	1.5607	2.7588	11.7833
10	0.6503	1.0049	1.4918	2.5549	10.8656
100	0.6267	0.8644	1.3594	2.0091	9.39750

Table 2. Minimum dwell time, T_d results of Lemma 2.

$H \setminus \lambda$	0	0.2	0.4	0.6	0.8
1	0.8538	1.2333	1.8882	3.2652	7.6227
2	0.7460	1.1248	1.6536	2.7175	6.0128
5	0.6727	1.0504	1.5150	2.4186	5.2460
10	0.6503	1.0039	1.4649	2.2631	5.0168
100	0.6267	0.8643	1.3450	1.9564	4.7364

Example 2 (Controller Design)**:** We consider the system (1) with A_1 and A_2 same as in Example 1 and

$$B_1 = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, \quad B_{w,1} = \begin{bmatrix} 0.1 \\ -2 \end{bmatrix}, \quad B_{w,2} = \begin{bmatrix} -0.5 \\ 0.2 \end{bmatrix}, \tag{15}$$

where the uncertainty is assumed to be $|\lambda| \leq 0.8$. We seek the uncertain system (14) that satisfies the same dwell time as in the nominal system (when the uncertainty, $\bar{\lambda} = 0$). Therefore, this can be achieved using the state feedback controller design Theorems 1 and 2.

To apply the robust controller design theorems in Section 3, the system is defined as:

$$C_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \\ 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \\ 0 & 0 \end{bmatrix}, \quad D_{u,1} = \begin{bmatrix} 0 \\ 0 \\ 0.12 \end{bmatrix}, \quad D_{u,2} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}, \tag{16}$$

with $H = 1$ and $T_d = 0.63s$, Theorem 1 gives

$$L_{1,1} = \begin{bmatrix} 61.7532 & -27.0538 \end{bmatrix}, \quad Q_{1,1} = \begin{bmatrix} 11.11 & -8.5247 \\ * & 56.4358 \end{bmatrix},$$

$$L_{1,2} = \begin{bmatrix} 70.1748 & -35.5474 \end{bmatrix}, \quad Q_{1,2} = \begin{bmatrix} 6.006 & -3.0581 \\ * & 50.6008 \end{bmatrix},$$

$$L_{2,1} = \begin{bmatrix} 20 & -9.9994 \end{bmatrix}, \quad Q_{2,1} = \begin{bmatrix} 6.1761 & -4.0448 \\ * & 56.323 \end{bmatrix},$$

$$L_{2,2} = \begin{bmatrix} 19.9998 & -10 \end{bmatrix}, \quad Q_{2,2} = \begin{bmatrix} 5.8495 & -3.8761 \\ * & 52.328 \end{bmatrix}.$$

The upper bound on the closed-loop H_2-gain is $\gamma = 0.2813$ and the time varying switched state feedback controller is computed with (12).

Using Theorem 2, with $H = 1$, $\beta = 0.04$ and $T_d = 0.63s$, we get $K_1 = \begin{bmatrix} 4.3634 & 0.373 \end{bmatrix}$, $K_2 = \begin{bmatrix} 0.9495 & 0.0337 \end{bmatrix}$, for which the upper bound on the closed-loop H_2-gain is $\gamma = 0.2743$.

The system is simulated with a periodic switching sequence $(t, i) = \{(0, 2), (T_d, 1), (2T_d, 2), \ldots\}$. Here, the parameter uncertainty is chosen as $\lambda = 0.8$. The responses of the open loop switched system are given in Figure 1 which shows that the system is unstable. Here, x1 and x2 represent the states of the system. Figure 2 presents the trajectory responses of the closed-loop (controlled) switched systems with respectively a time varying and constant controller gains that are asymptotically stable.

As it can be seen, the proposed control strategies can stabilize an uncertain switched system with a given dwell time. The constant controller is slower but has a smaller H_2-gain bound than the time varying controller.
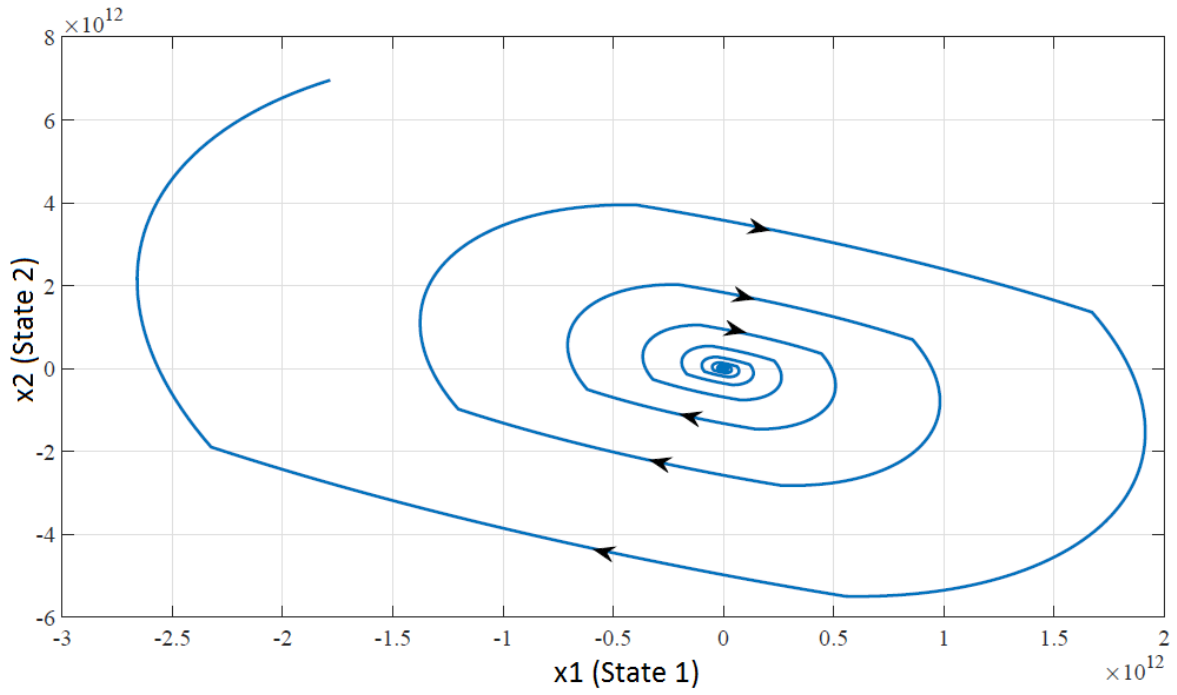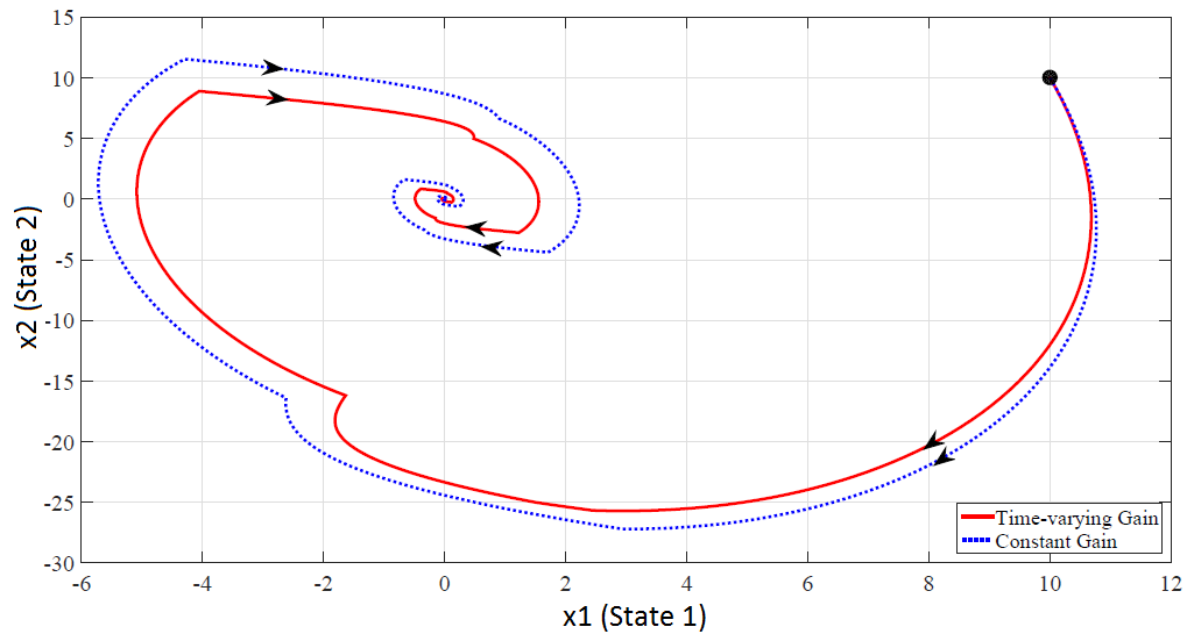
Figure 1. Open-loop response

Figure 2. Closed-loop responses.

5. Conclusion

In this paper, an H_2 sub-optimal state feedback controller synthesis method for switched linear systems with dwell time is presented. A time varying Lyapunov function and a parameter dependent Lyapunov function have been used to design a switched state feedback controller with H_2 bound guarantees. These approaches have been applied to illustrative examples.

In our future research, we intend to apply the proposed controller design strategies to more complex systems.

References

[1]　Liberzon D, Morse AS. Basic problems in stability and design of switched systems. Control Systems IEEE 1999; 19 (5): 59-70.

[2]　Daafouz J, Riedinger P, Iung C. Stability analysis and control synthesis for switched systems: A switched Lyapunov function approach. IEEE Transactions on Automatic Control November 2002; 47 (11): 1883-1887.

[3]　Goncalves M, Megretski A, Dahleh A. Global analysis of piecewise linear systems using impact maps and surface Lyapunov functions. IEEE Transactions on Automatic Control December 2003; 48 (12): 2089-2106.

[4]　Dayawansa P, Martin F. A converse Lyapunov theorem for a class of dynamical systems which undergo switching. IEEE Transactions on Automatic Control April 2004; 44 (4): 751-760.

[5]　Boyarski, S., Shaked, U., 2009, Time-convexity and time-gain-scheduling in finite-horizon robust H_∞-control, *Proceedings of the 48th IEEE Conference on Decision and Control, 2009 held jointly with the 28th Chinese Control Conference. CDC/CCC*, China, 2765-2770.

[6]　Allerhand LI, Shaked U. Robust stability and stabilization of linear switched systems with dwell time. IEEE Transactions on Automatic Control 2011; 56 (2): 381-386.

[7]　Allerhand LI, Shaked U. Robust state-dependent switching of linear systems with dwell time. IEEE Transactions on Automatic Control 2013; 58 (4): 994-1001.

[8]   Allerhand LI, Shaked U. Robust Control of Linear Systems via Switching. IEEE Transactions on Automatic Control 2013; 58 (2): 506-512.

[9]   Geromel JC, Colaneri P. Stability and Stabilization of Continuous-time Switched Linear Systems. SIAM Journal on Control and Optimization 2006; 145 (5): 1915-1930.

[10]   Allerhand, LI., Shaked, U., 2010, Robust control of switched linear systems with dwell time, *26th Convention of Electrical and Electronics Engineers in Israel (IEEEI)*, Israel, 198-201.

[11]   DeCarlo RA, Branicky MS, Pettersson S, Lennartson B. Perspectives and results on the stability and stabilizability of hybrid systems. Proceedings of the IEEE 2000; 88 (7): 1069-1082.

[12]   Sun Z, Ge SS. Stability theory of switched dynamical systems. Springer Science & Business Media; 2011.

[13]   Lin H, Antsaklis PJ. Stability and Stabilizability of Switched Linear Systems: A Survey of Recent Results. IEEE Transactions on Automatic control 2009; 54 (2): 308-322.

[14]   Liberzon D. Switching in Systems and Control. Systems & Control: Foundations & Applications Boston: Birkhäuser; 2003.

[15]   Zhao X, Zhang L, Shi P, Liu M. Stability of switched positive linear systems with average dwell time switching. Automatica June 2012; 48 (6): 1132-1137.

[16]   Hespanha, J., Morse, A., 1999, Stability of switched systems with average dwell time, *Proceeding of the 38th Conference on Decision and Control,* Phoenix, AZ, USA, 2655-2660.

[17]   Zhai, G., Hu, B., Yasuda, K., Michel, A., 2000, Stability analysis of switched systems with stable and unstable subsystems: an average dwell time approach, *Proceeding of the American Control Conference*, Chicago-IL-USA, 200-204.

[18]   Levine WS. Control System Advanced Methods. USA: Taylor and Francis Group; 2011.

[19]   Allerhand LI, Shaked U. Robust estimation of linear switched systems with dwell time. International Journal of Control Taylor & Francis 2013; 86 (11): 2067-2074.

[20]   Chesi, G., Colaneri, P., Geromel, J. C., Middleton, R., Shorten, R., 2010, Computing upper-bounds of the minimum dwell time of linear switched systems via homogeneous polynomial Lyapunov functions, *American Control Conference (ACC) 2010 IEEE*, Baltimore-MD-USA, 2487-2492.

[21]   Scherer C, Weiland S. Linear matrix inequalities in control Lecture notes for a course of the Dutch institute of systems and control. Delft University of Technology 2004.